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Gee Tsang

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CONCENTRATION OF EFFLUENTS IN A PLUME AS PREDICTED BY A MODEL AND OBSERVED IN FIELD

by

Gee Tsang

Fluid Mechanics Laboratory

Department of Mechanical Engineering
Massachusetts Institute of Technology

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ABSTRACT

Field-measured data of sulphur dioxide SO_2 and sulphur hexafluoride SF_6 concentrations in a plume are compared with theoretical predictions based on a plume rise model. The comparison shows that the theoretical model does correctly predict the concentration of pollutants in a plume. The comparison of the concentrations of SO_2 and SF_6, which was an inert gas released to the stacks as a tracer, shows that in the experimental range of 20 miles, the depletion of SO_2 is not an important factor in estimating the rate of dilution.

1. Introduction.

For the prediction of plume rise, a theoretical model based on the entrainment concept and the assumption of a "top-hat" distribution profile has been proposed by Hoult et al (1968). Using this model, the variation of concentration of an effluent along a plume can be predicted. The prediction of effluent concentration in a plume by this model had not been compared with field observations and this will be done in this paper. The concentration of effluents in a hot plume from smoke stacks has been sampled by Berger et al (1968) and Dennis et al (1969). Their field data will be used to provide field measured concentrations.

Today, it is of general belief that after leaving the stack SO_2 will deplete from the plume as a result of physical, chemical and photochemical reactions with the atmosphere. One of the more accepted theories considers that SO_2 depletes from the plume exponentially with time. The half-life time of SO_2, which is the time for SO_2 to reduce to half its concentration, depends on the atmospheric conditions and is considered to vary from a couple of hours to a few days. From their studies, Berger et al and Dennis et al considered that the half-time of SO_2 in a plume is of the order of an hour. Using their data, a study on the dilution of SO_2 from a plume will be made and it will be shown that the depletion of SO_2 is not an important factor in affecting the estimate of the concentration of SO_2 by the theory in the test range of 20 miles.

2. Theory.

The plume rise model proposed by Hoult et al predicts that the centre line of a hot plume in a neutral atmosphere or in a stably stratified atmosphere while the plume is still rising is given by

$$z = \left(\frac{3}{2\beta^2}\right)^{1/3} \ell_b^{1/3}\, x^{2/3} \qquad (1)$$

In the above equation, ℓ_b is a buoynacy length scale defined by

$$\ell_b = \frac{u_i b_i^2 (\frac{T_i - T_\ell}{T_\ell}) g}{v^3} \qquad (2)$$

The meaning of other symbols is: z = height above the stack exit, x = distance downwind from stack, u_i = gas velocity at the stack exit, b_i = radius of the stack exit, T_i = absolute temperature of flue gas at the stack exit, T_ℓ = absolute temperature of the atmosphere at the stack exit, v = cross wind velocity, β = entrainment coefficient and g = gravitational acceleration. Equations of a similar form to equation (1) are also obtained by other researchers from different approaches (Briggs (1965), Slawson and Csanady (1967)). The theoretical model shows that in the range where equation (1) applies, the radius of the plume is given by

$$b = \beta z \qquad (3)$$

A study of β from field observations has been made by Fay et al (1969). They found that β varies randomly from 0.3 to 4.3 with a median of 0.81 and when $\beta = 0.9$ the rms value of the difference between the measured plume rise and the computed plume rise is minimal.

The theoretical plume rise model may be used for the prediction of concentration. If the concentration of an effluent at the stack exit is c_i and at at a distance x from the stack is c, then from the conservation of mass one has

$$\pi b_i^2 c_i = \pi b^2 c \qquad (4)$$

The combination of the above equation with equations (1) and (3) gives

$$\left(\frac{c}{c_i}\right)_{\text{theory}} = 0.7633^{-2/3} \frac{u_i}{v} \left(\frac{b_i}{\ell_b}\right)^2 \left(\frac{x}{\ell_b}\right)^{-4/3} \qquad (5)$$

which means that knowing the plume and atmospheric characteristics, the concentration of an effluent in a plume can be calculated. It should be noted that equation (5) is obtained assuming that there is no depletion of the effluent. It is seen from equations (2) and (5) that under given plume characteristics, the concentration at a fixed downwind point is proportional to the wind velocity. Physically, this means that a plume becomes slenderer in a higher crosswind and consequently gives a higher concentration at a fixed point.

3. Field observation of concentration in a smoke plume.

The field data collected by Berger et al and Dennis et al were the atmospheric characteristics, the plume characteristics, and the concentration of sulphur dioxide SO_2 and sulphur hexafluoride SF_6 in a plume from a power station situated by the seashore. SF_6 is an inert gas and was released to the flue gases as a tracer. The sampling was made by an airplane criss-crossing the plume at various downwind distances and at various altitudes. To avoid the effect of ground roughness, sampling was only made when the plume was blown offshore to the sea. The stretch of the plume being sampled was from 1/4 miles to 20 miles. Although the height of the plume at the distance of 20 miles was not recorded, the recorded data showed that in the range from 1.75 miles to 12 miles the plume was in a continuous rise. Therefore it might be expected that in the whole experimental range of from 1/4 to 20 miles the plume was rising and a comparison between the observed concentration and the theoretical concentration as given by equation (5) is proper.

The plume boundary was determined by an airborne electro-conductivity analyzer which could respond to an increment in SO_2 level above the background level within 5 seconds. Having established the plume boundary,

several traverses were made across the plume for the sampling. The concentration of SO_2 was obtained by two ways. The first was the "on the spot" analysis by which flue gases were drawn from the plume to pass a bubbling train installed on the airplane. The second was to draw the plume gases into Saran bags during the traverse for later laboratory determination of the concentration. For the concentration of SF_6, only the second way was used. The recorded data showed that in the test range, the width of the plume was roughly between 1/2 miles to 1-1/2 miles. For the light aircraft used for the sampling, if an air speed of 90 m.p.h. is assumed, the time required for making a traverse was of the order of 2/3 minutes.

4. Comparison between theory and observations.

The theory discussed in 2. is for plumes from a single smoke stack while on the other hand the power station being studied had three smoke stacks. To make a comparison between the theory and the field measurement feasible, an equivalent stack is assumed to exist which emits the same mass, momentum and heat flux as emitted by the three stacks. Using these conditions, the temperature of the gas, the exitting velocity and the radius of the stack exit of the equivalent stack may be calculated. The location of the equivalent stack is assumed to be co-incident with the middle of the three stacks. The introduction of such an equivalent stack will not introduce discrepancies at distances not too close to the stacks, since Dennis et al showed that at a distance of 1.75 miles downwind from the stacks the width of the plume was about 3000 feet, which is large comparing to the separation of two stacks of 50 feet. However, at distances close to the stacks, large errors should be expected. The field measurement would give a lesser concentration since the mixing surface area of the three composing plumes is larger than that of the

equivalent plume for dilution.

Knowing the temperature and velocity of the exitting gas and the radius at the exit of the equivalent stack, equations (2) and (5) may be used for calculating the theoretical concentration. Equation (5) shows that for plumes of the same parameter $\frac{u_i}{v}\left(\frac{b_i}{\ell_b}\right)^2$, the plotting of $\frac{c}{c_i}$ versus the normalized distance $\frac{x}{\ell_b}$ in a log-log scale would be linear with a slope of -4/3. To examine this relationship, field data of three days for the measurement of the concentration of SO_2 by Dennis et al are used. (The parameter $\frac{u_i}{v}\left(\frac{b_i}{\ell_b}\right)^2$ for the plumes in these three days was almost the same). The plotting is shown in Figure 1. Although the plotting shows a certain scattering, a straight line with a slope of -4/3 can be drawn to fit the points. This straight line is shown by the dashed line in the figure. The theoretical straight line of $\frac{c}{c_i}$ versus $\frac{x}{\ell_b}$ is also shown in Figure 1 by the solid line. The theoretical curve is calculated using the least rms error value of β of 0.9. For all theoretical calculations in this paper, this value of β will be used.

Owing to the turbulent nature of the plume motion, the random variation of β and the inaccuracy of the data-collecting method, the scattering of the data points or the deviation of the dashed line from the solid line should be expected. However, as it is seen from Figure 1 that although the dashed line is drawn for data of only three days, the deviation of it from the solid line is small. It may be presumably correct to expect that if more field data are available, the dashed line will gradually shift towards the theoretical curve. Thus, the field measurement showed that the concentration of an effluent $\frac{c}{c_i}$ is indeed

proportional to $(\frac{x}{\ell_b})^{-4/3}$ as predicted by equation (5).

From the field data by Dennis et al the theoretical concentration of SO_2 may be calculated and compared with the field-measured concentration by plotting $(\frac{c}{c_i})_{measured}$ versus $(\frac{c}{c_i})_{theory}$ as shown in Figure 2. When the measured concentration equals the theoretical concentration, the data points fall on the 45° line. It is seen from Figure 2 that all the data points fall in the neighbourhood of the 45° line. The scattering of the points is within the same order of magnitude and indicates that $(\frac{c}{c_i})_{measured} \simeq (\frac{c}{c_i})_{theory}$ or $(c)_{measured} \simeq (c)_{theory}$. A higher degree of scattering of the data points is noted for the range of higher concentration. This is in fact expected since it is mentioned earlier in this section that at distances close to the stacks the equivalent stack concept gradually loses its effectiveness and the theory would give a higher concentration than actual. The above is indeed observed in Figure 2 as the points for $(\frac{c}{c_i})_{theory}$ greater than 2×10^4 appear to scatter to the underside of the 45° line.

Similar to Figure 2, a plotting of $(\frac{c}{c_i})_{measured}$ versus $(\frac{c}{c_i})_{theory}$ for the inert tracer gas SF_6 can also be obtained as shown in Figure 3. It should be noted that the initial concentration of SF_6 at the stack exit is calculated from the mass rate of release of SF_6 to the flue gas in Figure 3 while in Figure 2 the initial concentration of SO_2 was measured at the stack exit. It is seen from Figure 3 that the same conclusions as for the case of SO_2 may be drawn.

A study of Figures 1, 2 and 3 shows that there is a general tendency for the measured concentration to be less than the theoretical concentration. This may be mainly a reflection of the experimental method which tends to

give a lower concentration than the true value in the plume. There are three reasons for this underestimation. The first is the traverse sampling. It is mentioned in 3 that samples were obtained by drawing gases from the plume while the airplane was making traverses. Since the real distribution of pollutant in a plume is bell-shaped, a true mean can only be obtained if the plane of traverse contains the peak, otherwise the sample will show a lower average. It is easy to imagine that the sampling airplane used by Dennis et al would more likely than not miss the peak and thus give a lower mean. The second reason for underestimation is the response time of the electro-conductivity analyzer. The analyzer was switched on while the plane was criss-crossing the plume making a sampling. From the recording of the analyzer, the variation of concentration of the sampled gases with time and consequently the fraction of the sampling time in which the plane would be in the plume was obtained. Knowing the "in plume" time, the proportion of flue gases to ambient gases in the gas sample could be calculated, and this in turn enabled the concentration of the pollutants in the plume to be computed. It was pointed out by Bergers et al that the response time (5 seconds in the present case) has an effect of spreading the concentration-time curve and thus gives a longer "in plume" time than actually happened. This overestimate of "in plume" time in turn gives an underestimate of pollutant concentration. The third reason for underestimation is the effect of the sampling time. It is mentioned in 3. that the time required for the airplane to make a traverse was of the order of 2/3 minutes. In this time the plume would have not remained in the same course but swept through a certain angle, thus the samples obtained were not instantaneous samples of the plume but rather time-averaged samples. According to the theory of diffusion, the time-averaged concentration is lower than the instantaneous concentration.

Figures 1, 2 and 3 make use of equation (5) which is obtained assuming that there is no loss of pollutant to the atmosphere. SF_6, being an inert gas, satisfies this "no loss" condition and the use of equation (5) for it is justified. SO_2, on the other hand, is generally believed to deplete from a plume due to chemical and photo-chemical reactions with the atmosphere and the use of equation (5) for it is subject to some doubt. However, the similarity between Figures 2 and 3 shows that in the experimental range SO_2 behaved the same as SF_6 and therefore the depletion of SO_2 from the plume should be insignificant. The agreement between the field data points and the theoretical curve in Figure 1 further strengthens this point.

The depletion of SO_2 can be detected by comparing the dilution of SO_2 $(c/c_i)_{SO_2}$ with the dilution of SF_6 $(c/c_i)_{SF_6}$ at the same point. For this reason the ratio of $(c/c_i)_{SO_2}$ / $(c/c_i)_{SF_6}$ is plotted against

$\beta^2 \frac{v}{u_i} \left(\frac{\ell_b}{b_i}\right)^2 \left(\frac{x}{\ell_b}\right)^{4/3}$ (see equation (5)) as shown in Figure 4. It is seen from Figure 4 that the scattering of the data points does not lead to a clearly defined functional depletion of SO_2 from the plume. In fact the data points appear to scatter about the horizontal line of $(c/c_i)_{SO_2}$ / $(c/c_i)_{SF_6} = 0.84$ or $(c/c_i)_{SO_2}$ / $(c/c_i)_{SF_6} = 1$ seeing that the estimate of the quantity of SF_6 was subject to an error of up to 33 percent (Dennis, private communication), and this means that in the test range of 20 miles, the comparison shows no significant depletion of SO_2. It is of Dennis' opinion (private communication) that the laboratory analysis of the 12 miles point samples,

or the $\beta^{2/3} \frac{v}{u_i} \left(\frac{\ell_b}{b_i}\right)^2 \left(\frac{x}{\ell_b}\right)^{4/3} \approx 3 \times 10^4$ data points, might be erroneous.

If these 5 points and also the marked point, which is suspected to have a decimal error, are excluded from Figure 4, the scattering of the data points will be greatly reduced. However, even under this condition, the scattering of the data points still does not lead to a clearly defined functional depletion of SF_6. The data points still appear to fluctuate about a horizontal line of

$(c/c_i)_{SO_2} / (c/c_i)_{SF_6} = \text{constant}$.

Conclusion:

From the above discussions the following conclusions may be made:

(1) The plume rise model proposed by Hoult may be employed for the prediction of concentration of a pollutant in a plume. The concentration varies with downwind distance to the -4/3 power.

(2) In the experimental range of 20 miles, the depletion does not significantly affect the concentration of SO_2.

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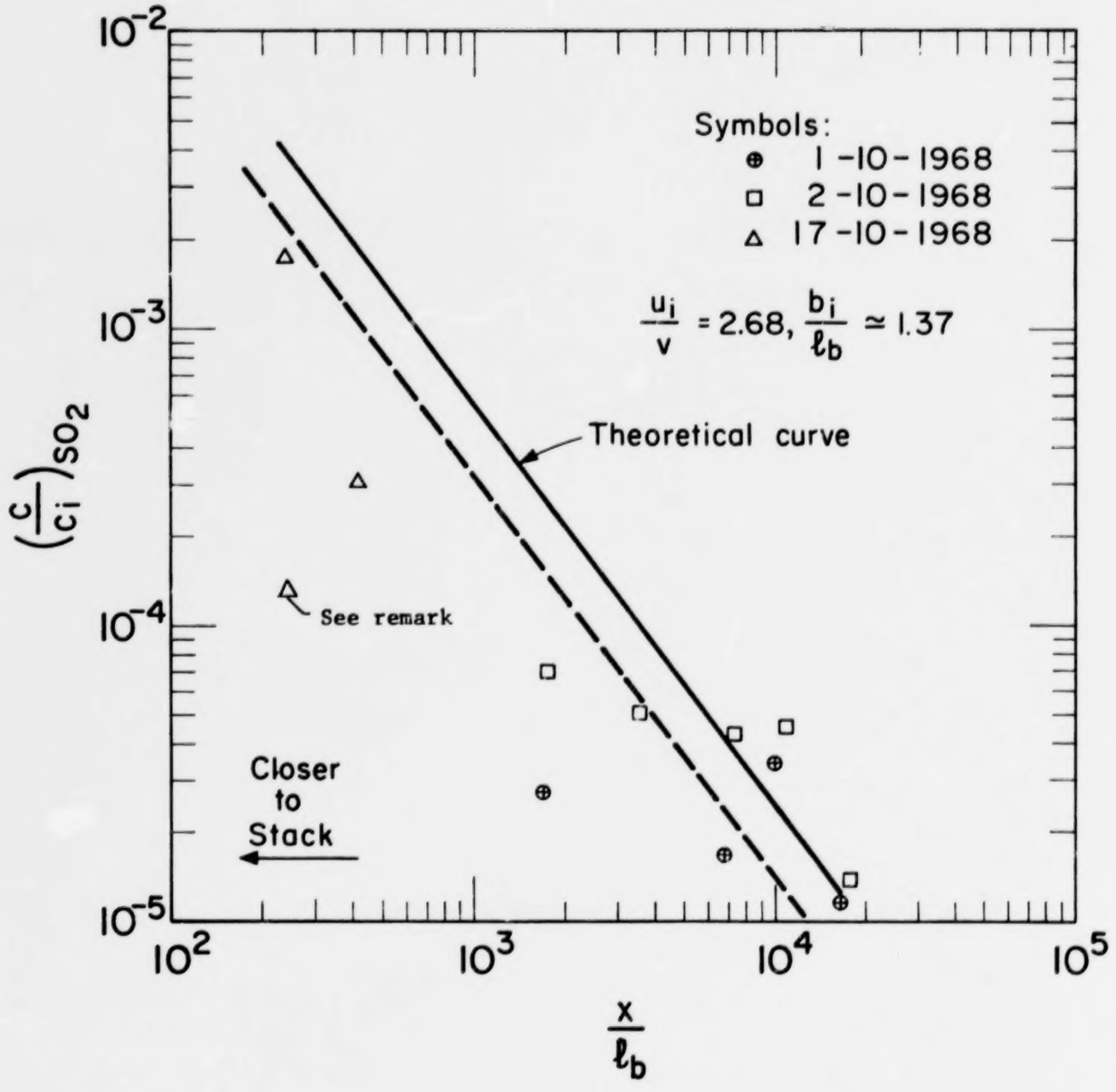

Fig. 1. Variation of concentration of SO_2 with normalized distance downwind.

((Remark; Decimal error suspected for the marked point)

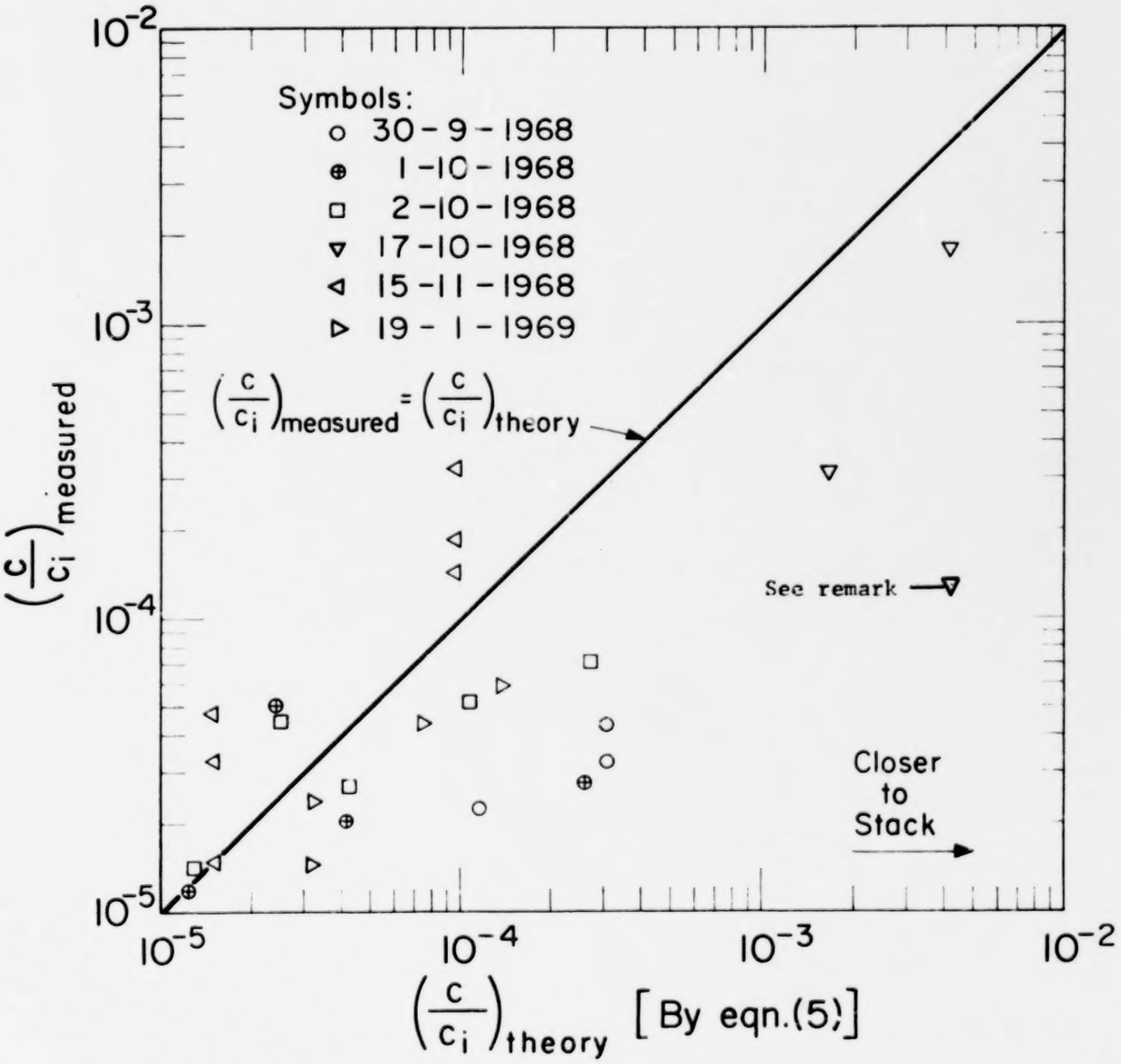

Fig. 2. Comparison of SO_2 concentration as predicted by theory and measured in field.

(Remark; Decimal error suspected for the marked point)

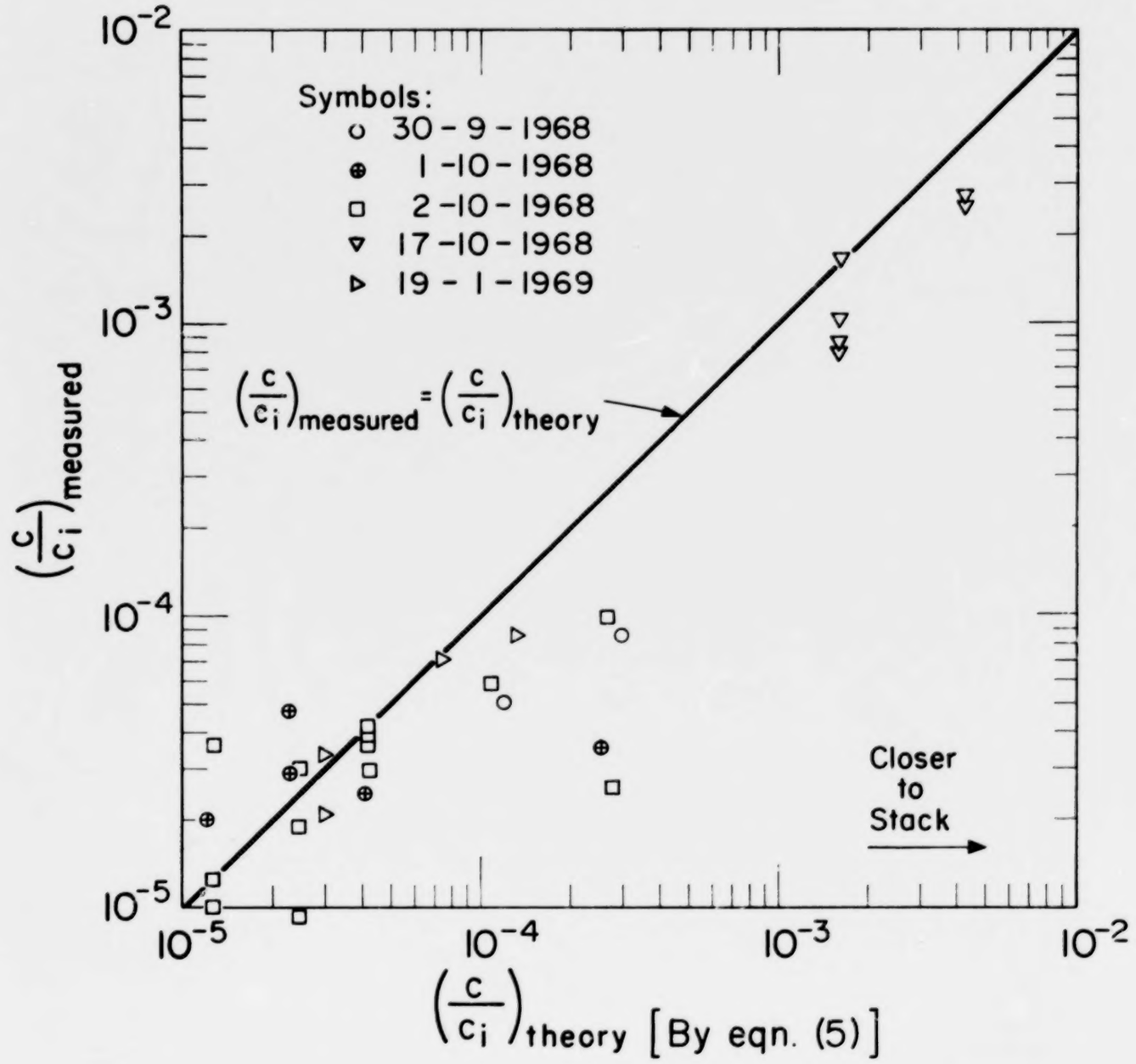

Fig. 3. Comparison of SF_6 concentration as predicted by theory and measured in field.

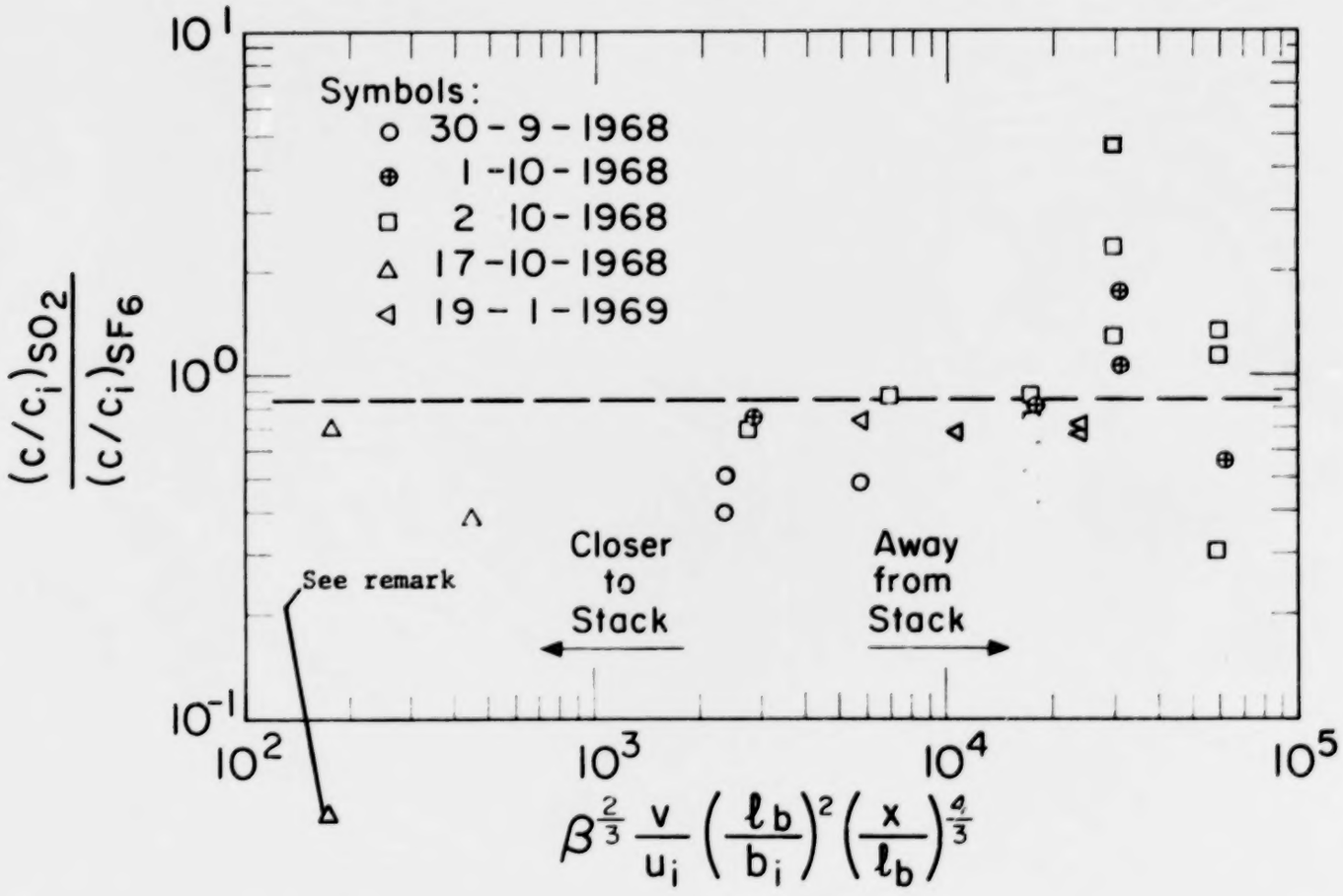

Fig .4 Variation of the ratio of dilution of SO_2 to SF_6

(Remark; Decimal error suspected for the marked point)

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